Searching for Patterns in Random Sequences

George Wolford, Sarah E. Newman, Dartmouth College
Michael B. Miller, University of California at Santa Barbara and Dartmouth College
Gagan S. Wig, Dartmouth College

Abstract In a probability-guessing paradigm, participants predict which of two events will occur on each trial. Participants generally frequency match even though frequency matching is nonoptimal with random sequences. The optimal strategy is to guess the most frequent event, maximizing. We hypothesize that frequency matching results from a search for patterns, even in random sequences. Using both callisotomy patients and patients with frontal brain damage, Wolford, Miller, and Gazzaniga (2000) found frequency matching in the left hemisphere but maximizing in the right hemisphere. In this paper, we show that a secondary task that competes for left hemisphere resources moves the participants toward maximizing but that a right-hemisphere task preserves frequency matching. We also show that a misunderstanding of randomness contributes to frequency matching.

People experience difficulty in estimating and understanding uncertainty in many situations (Kahneman, Slovic, & Tversky, 1982). We believe that the search for patterns is an important factor in people's behaviour in uncertain situations. An examination of people's behaviour in a probability-guessing task may help us understand the way they deal with uncertainty. We provide evidence from probability-guessing experiments that people look for patterns in sequences of events, even when informed that there are no patterns and that this search is related to their understanding of uncertainty.

In a standard probability-guessing situation, there are at least two stimuli (e.g., lights, optimal routes to work, etc.) that are presented on some schedule. With two stimuli, the probabilities of the two stimuli are p and (1-p). We are most interested in the case in which p is greater than .50. Participants are asked to predict which stimulus will be presented on each trial. Frequency matching refers to guessing the most frequent stimulus with probability p. In a case in which $p = .70$, frequency matching yields correct answers 58% of the time (0.7·0.7 + 0.3·0.3). Maximizing, always guessing the stimulus that has the highest probability

of appearing, yields correct answers 70% of the time (0.7·1.0 + 0.3·0.0). Maximizing is superior to frequency matching as long as p ≠ .5 and the sequence is truly random.

When asked to predict which of two things will occur over a series of trials, most humans exhibit frequency matching. That is, they make predictions in proportion to the frequency with which the events have occurred in the past (Estes, 1961). Frequency matching has intrigued scientists since the 1930s (Humphreys, 1939) for at least two reasons: It is a nonoptimal strategy for random sequences and most other species exhibit maximizing, the optimal strategy (Hinson & Staddon, 1983). We propose that people exhibit frequency matching because they look for patterns in sequences even when told that the sequences are random. If there were a real pattern in the data, then any successful hypothesis about that pattern would result in frequency matching. If people were searching for patterns, then frequency matching would not be a strategy, per se, but a consequence of the search for patterns.

Yellott (1969) provided a striking demonstration of the extent to which people seek patterns in sequences. In his experiment, a light was flashed to either the left or the right on each trial and subjects had to predict which light would appear. Participants experienced many trials and p varied across blocks. Subjects' predictions matched the frequency of the actual presentations (frequency matching), changing when the frequency changed. In the final 50-trial block, the contingencies changed without the participants' knowledge. The light appeared wherever the subject predicted it would. In other words, if the subject guessed left, it was left, etc. Subjects continued to frequency match during these last 50 trials. When Yellott stopped the experiment and asked subjects for their impressions, they overwhelmingly responded that there was a fixed pattern to the light sequences and that they had finally figured it out. They proceeded to describe elaborate sequences of right and left choices that resulted in their responses always being correct. These verbal reports support the contention that subjects had been

searching for fixed sequences all along and were fooled into thinking they had succeeded.

Wolford, Miller, and Gazzaniga (2000) provided evidence that this search for patterns might be occurring in the left hemisphere. They reasoned that the search for patterns might be related to a proposed neural module, the interpreter, that was proposed by Gazzaniga and his colleagues (Gazzaniga, 1989, 1995; Metcalfe, Funnell, & Gazzaniga, 1995). The interpreter is thought to play the role of trying to make sense out of the information that it confronts (Gazzaniga, 1989, 1995; Metcalfe et al., 1995). Using split-brain patients, who have had their corpus callosum severed to treat intractable epilepsy thus preventing interhemispheric transfer of information at the cortical level, Gazzaniga (1989) provided evidence that the interpreter is located in the left hemisphere. To test our hypothesis about the relationship between searching for patterns and the left hemisphere, we carried out a probability-guessing paradigm in two split-brain patients and in several patients with unilateral damage to the frontal region of one of the hemispheres. We hypothesized that if processes housed in the left hemisphere were responsible for the search for patterns, then we should observe frequency matching in the left hemisphere and maximizing in the right hemisphere. That is what we found. The patients with unilateral cortical damage replicated the findings with the split-brain patients.

In the present paper, we use competing-task paradigms in participants with intact brains to corroborate the previous findings with split-brain patients. We used one competing task that we presumed would engage primarily left-hemisphere resources and one that we presumed would engage primarily right-hemisphere resources. Based on our previous work, we predicted that the left-hemisphere task would interfere with frequency matching but the right-hemisphere task would not.

In a second experiment, we capitalized on people's misunderstanding of random sequences (Falk & Konold, 1997). We altered the conditional probabilities between events to create sequences that conform to peoples' common, but incorrect, views about randomness to see if those sequences yield different patterns of behavior than truly random sequences.

Experiment 1

In Experiment 1, participants engaged in a probability-guessing task in the presence of distracting tasks. Our previous research suggested that the neural processes responsible for frequency matching are in the left hemisphere. Therefore, we wanted one distracting task that would compete primarily for left-hemisphere processes and a second distracting task

that should compete primarily for right-hemisphere processes. Our prediction was that subjects would continue to frequency match in the presence of a right-hemisphere distracting task but would maximize with a left-hemisphere distracting task.

For the left-hemisphere task we chose a variant of the three-back verbal working memory task. Results from neuro-imaging studies show asymmetric activation of structures in the left hemisphere for verbal working memory tasks (Awh et al., 1996; Coull, Frith, Frackowiak, & Grasby, 1996). Dunbar and Sussman (1995) used a similar verbal working memory task to simulate frontal-brain damage in participants without damage. Participants in the verbal working memory group had to maintain three digits in memory at all times while guessing whether the target would occur at the top or bottom of the screen. Our prediction was that the participants who had to engage in this distracting task would be unable to devote the resources required to look for patterns and might switch to maximizing.

For the right-hemisphere task, we had participants engage in a same/different or one-back task using randomly constructed polygons. Results from neuro-imaging studies show asymmetric activation of structures in the right hemisphere for visual-spatial working memory tasks (Smith et al., 1995).

Method

Participants

Thirty Dartmouth undergraduates served in the first experiment, 10 in each of three conditions. Subjects received credit toward the introductory psychology course as well as the possibility of monetary reward based on performance.

Design and Procedure

Participants were randomly assigned to one of three distractor-task conditions: a verbal working-memory task, a visual-spatial working-memory task, or a no-distractor task. The distractor task was interwoven with a probability-guessing paradigm in which the more frequent choice appeared 75% of the time, and the less frequent 25%. Participants were informed that the target in the probability-guessing task was chosen randomly. They were paid $.01 for each correct prediction and were instructed to be as accurate as they could on both tasks.

Verbal working-memory task. The verbal working-memory task required subjects to remember the last three digits they saw. Each trial began with the presentation of a digit in the middle of the screen. The digit served as a cue to make a guess in the probability-
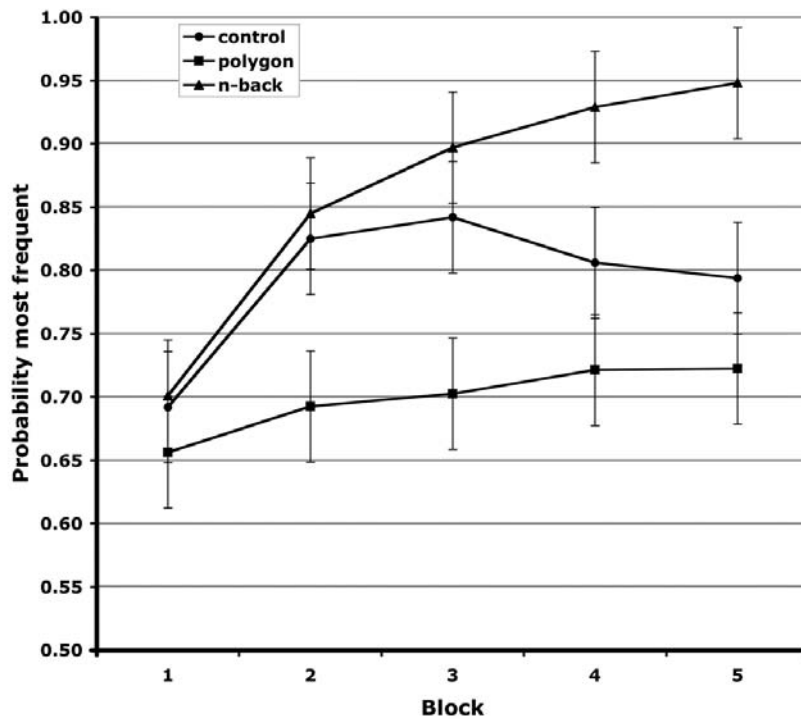
Figure 1. Probability-guessing behaviour as a function of the presence and type of competing task in Experiment 1. Points represent the probability of choosing the most frequent alternative. Error bars represent standard errors.

guessing task. After each prediction, a new digit appeared. Participants were instructed to maintain the last three digits in memory, updating the set with the appearance of each new digit. Each digit was chosen randomly from the set of 10 digits. Participants were randomly probed four times per block and asked to type in the last three digits they had seen. Participants were told that errors on the memory task would reduce the earnings on the probability-guessing task.

Visual-spatial task. Each trial of the visual-spatial task began with the presentation of a six-sided polygon in the centre of the screen. The participants had to judge whether the polygon was the same as or different than the one the trial before, by hitting the appropriate key. The polygon was a blue outline that had six vertices. The original polygon on Trial 1 was generated randomly, subject to certain size constraints, and centred on the screen. On each subsequent trial the polygon was altered randomly with probability .50 using the computer language's random function. On trials in which the polygon was altered, the change was accomplished by moving a single, randomly chosen vertex 30 pixels (680 x 480 screen) in one of the four cardinal directions (N, E, S, W). After responding

same or different to the polygon task, a 500-ms pause was inserted and then the participant made a prediction for the probability-guessing paradigm. Participants were told that errors on the polygon task would reduce the payoff from the probability-guessing task.

Guessing task. The experimenter advised the participants that one of their tasks was to predict whether a coloured square would appear above or below a fixation cross (+) presented in the middle of the screen. Participants were informed that the sequence of ups and downs was completely random. To indicate their prediction, the subjects would press the appropriate button on the keyboard. The correct buttons were labeled top or bottom. Upon predicting either top or bottom, a coloured square would then appear either on the top or bottom of the screen, providing immediate feedback regarding the accuracy of their prediction.

The experiment was carried out on iMacs with the screen resolution set to 640 x 480. The experiment consisted of five blocks of 100 trials. After every 100 trials the participants received additional feedback indicating the accuracy of their predictions and the accuracy on the distracting task for that block of 100 trials.

TABLE 1
Performance on the Secondary Tasks by Block

	Task	
Block	N-Back (s.e.)	Polygon (s.e.)
1	0.780 (0.079)	0.746 (0.029)
2	0.833 (0.071)	0.742 (0.025)
3	0.900 (0.067)	0.746 (0.019)
4	0.913 (0.066)	0.753 (0.035)
5	0.827 (0.070)	0.748 (0.027)

Results

The primary results are displayed in Figure 1. The points in the figure represent the probability of choosing the most likely alternative. Overall, the participants in the Control and Polygon Groups approached frequency matching and the participants in the 3-Back Group approached maximizing. The linear contrast for the interaction between Group and Block reflects the growing separation between the conditions in later blocks and was significant, $F(1,27) = 4.31$, $p = .02$. The omnibus interaction of Group and Block was significant with a multivariate $F(8,50) = 3.13$, $p = .006$. Using t-tests with Bonferroni corrections, the means of the 3-Back Group were significantly higher than the means of the other two groups during the last two blocks. The Control and Polygon Groups differed on Blocks 3 and 4, but the Polygon task was actually closer to frequency matching than the Control Group throughout. Finally, to look at maximizing in individual subjects, we defined maximizing as choosing the most frequent alternative 95% or more of the time in the last two blocks. Using that definition, zero participants maximized in the Control Group, one participant maximized in the Polygon Group, and seven participants maximized in the 3-Back Group. Those differences yield $\chi^2(2) = 14.66$, $p = .0008$.

Looking at performance on the secondary tasks, participants were slightly more accurate on the 3-Back task than the polygon task (.85 versus .75). That performance is shown by block in Table 1. The accuracy scores were quite variable in the 3-Back task, so the difference between tasks was not significant, $t(18) = 1.633$, $p = .12$. In neither task was accuracy on the competing task correlated with the degree of maximizing, yielding correlations of .14 and .09 between accuracy on competing task and performance on probability-guessing averaged over the final two blocks.

We interpret the results to indicate that participants continue searching for patterns in the presence of the polygon task, but participants do not do so in the presence of the 3-Back task.

Experiment 2

Our proposal is that people exhibit frequency matching because they are searching for patterns in the sequence even though they are told the sequence is random. The results of the first experiment supports that notion. We think that one reason people search for patterns in the face of contrary information is that random sequences do not look random. Several investigators have shown that when people are asked to construct random sequences they include too few long runs and too many alternations (Falk & Konold, 1997; Lopes, 1982; Lopes & Oden, 1987). We reasoned that if the sequences were constructed to look more like the average person's stereotype of a random sequence, then participants might abandon the search for patterns and move toward maximizing. In the third experiment, we made the sequences look "more random" by altering the conditional probabilities to break up runs.

Method

Participants

Thirty Dartmouth undergraduates served in the second experiment, 15 in each of two conditions. Subjects received credit toward the introductory psychology course as well as the possibility of monetary reward based on performance as in the previous experiments.

Design and Procedure

Participants were randomly assigned to one of two conditions: a control condition and an altered-conditional condition. In the control condition, we used the probability-guessing task described in Experiment 1 with no competing tasks. The most frequent alternative occurred on 70% of the trials.

For the control group, each trial was independent of the preceding one so the probability of the most frequent alternative was 70% independent of which stimulus appeared previously. In the altered conditional group, the probability of the most frequent stimulus on the first trial was .70. On all subsequent trials, the probability of the most frequent stimulus was conditionalized on the preceding trial. The probability of presenting the most frequent alternative was .60 if the most frequent one had just occurred versus .93 if the other alternative had just occurred. Those conditionals yield a marginal probability of .70 for the top, but the probability of an alternation is increased from .42 to .56 compared to the truly random condition in which the conditionals equal the marginals. The altered sequences were still "random" in the sense that the location of the next alternative was not perfectly predictable. Regardless of which condition they were assigned to, participants were told that the boxes would appear randomly at the top or the bottom.
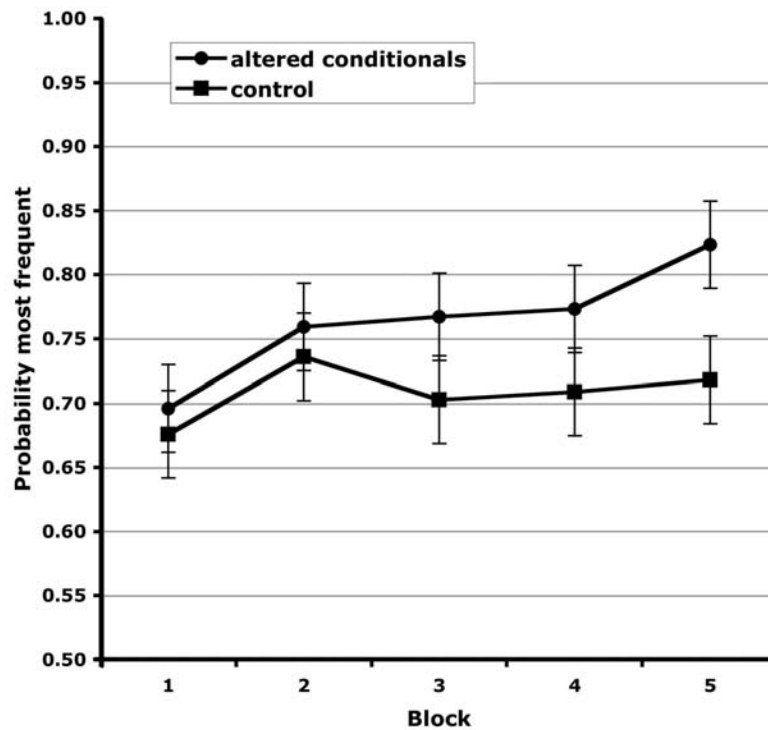
Figure 2. Probability-guessing behaviour for a control condition versus a condition in which the conditional probabilities have been altered to appear more random from Experiment 2. Error bars represent standard errors.

Results

The results are shown in Figure 2. We predicted that the group with altered conditionals would move toward maximizing, but the control group would frequency match as usual. Specifically, we predicted that the amount of difference between the two groups would increase across blocks. This prediction was tested and confirmed with a linear contrast on the interaction term. The contrast yielded an $F(1,28) = 4.04$, $p = .05$. The groups were significantly different by Block 5, $t(28) = 2.16$, $p = .04$.

The differences between the groups are even more striking if you look at the probability of responding with the most frequent stimulus on Trial N, conditionalized on which stimulus occurred on Trial N-1. Figure 3 shows the difference between those two conditionals for each of the two groups. As shown in Figure 3, participants in the Control Group are about 20% to 25% more likely to respond to the most frequent one after just seeing the most frequent one than after seeing the least frequent one. The actual probabilities of those two conditional probabilities are identical in the Control Group. In contrast, the participants in the Altered Conditional Group respond less differently as a function of the preceding stimulus, especially in the

first few hundred trials, even though there really is a 33% difference between the two conditionals in that group. The main effect of Group was highly significant, $F(1,28) = 17.68$, $p < .001$. Neither the main effect of Block nor the interaction of Block and Group was significant although the interaction approached significance, $F(4,66.2) = 2.379$, $p < .079$. To reiterate, in the group in which the conditionals are actually the same, participants predict a big difference, but in the condition in which they are very different, participants predict less difference and the difference goes in opposite directions in the two groups. This difference is consistent with our belief that participants see the sequences with altered conditionals as random and are less likely to look for patterns, but participants in the control group do not believe the sequences are random and therefore search for clues to uncover the "true" sequence.

General Discussion

Based on our previous research with both split-brain patients and patients with selective frontal lesions, we hypothesized that frequency matching in a probability-guessing task is the result of searching for patterns and that the search for patterns asymmetrical-
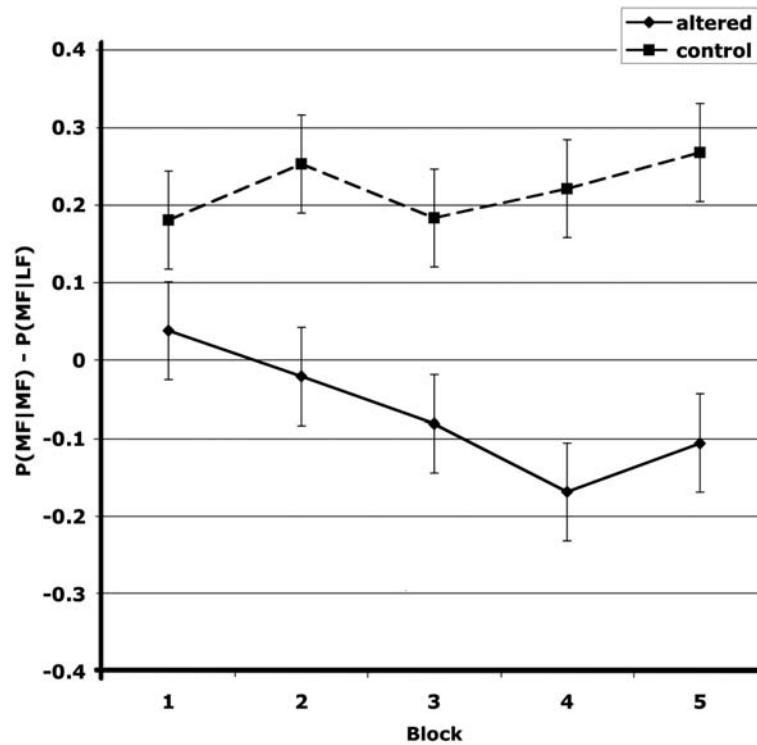
Figure 3. The probability of a "most frequent" response following a most frequent outcome on the preceding trial minus the probability of a "most frequent" response following a least frequent outcome on the preceding trial from Experiment 2. Error bars represent standard errors.

ly involves structures in the left hemisphere. By using competing tasks that drew differentially on the two hemispheres, we found some support for that earlier hypothesis. In Experiment 1, participants did maintain frequency matching when the competing task primarily involved right-hemisphere resources, but approached maximizing when the task primarily involved left-hemisphere resources.

Our proposal that participants regularly search for patterns is supported by the comments of the participants as in Yellott (1969). During debriefing, many of our subjects mentioned that they were searching for patterns.

In the second experiment we found support for the contention that people's misunderstanding of randomness contributes to the search for (and belief in) patterns. Specifically, those that received altered conditionals that conformed to the common (and incorrect) perception of randomness were found to approach maximizing.

Note that in an important sense, there was more predictability in the sequences with the altered conditionals. H is a measure of uncertainty or information (Attneave, 1959) computed according to Equation 1:

$$H = -\sum \pi \log_2 \pi \qquad (1)$$

We computed the value of H on pairs of trials for each condition. Pairs are required to capture the difference in conditionals. The value of H for normal sequences in which each trial is independent is 1.76 and the value of H for the group with altered conditionals is 1.67. Lower values of H indicate less uncertainty and thus more predictability. The maximal possible uncertainty with only two alternatives would be to have each one have an independent probability of .5 and that would yield an H of 2.0. The value of H or some similar measure is relevant because we are arguing that participants will give up the search for structure in the sequences with altered conditionals even though those sequences have more structure by objective measures.

Probability-guessing experiments tend to elicit fairly regular functions over blocks (Estes, 1961). Whether the subject is frequency matching or maximizing, he or she has to learn what the frequencies are. The subject has no information about frequencies in Trial 1. Therefore, the mean probability of guessing a particular outcome during that block represents some combination of learning about the frequencies and later strategies.

We believe that people are constantly searching for patterns in sequences. Such a search is of great utility

if there are patterns, but is counterproductive if there are not. We believe that the search for patterns in sequences that are truly random impacts the use and understanding of uncertainty, often to the detriment of human behaviour.

Correspondence should be addressed to George Wolford, Department of Psychological and Brain Sciences, Dartmouth College, 6207 Moore Hall, Hanover, NH 03755 (E-mail: george.wolford@dartmouth.edu).

References

Attneave, F. (1959). *Applications of information theory to psychology*. New York: Holt, Rinehart and Winston, Inc.

Awh, E., Jonides, J., Smith, E. E., Schumacher, E. H., Koeppe, R. A., & Katz, S. (1996). Dissociation of storage and rehearsal in verbal working memory: Evidence from positron emission tomography. *Psychological Science*, *7*, 25-31.

Coull, J. T., Frith, C. D., Frackowiak, R. S. J., & Grasby, P. M. (1996). A fronto-parietal network for rapid visual information processing: A PET study of sustained attention and working memory. *Neuropsychologia*, *34*, 1085-1095.

Dunbar, K., & Sussman, D. (1995). Toward a cognitive account of frontal lobe function: Simulating frontal lobe deficits in normal subjects. *Annals of the New York Academy of Sciences*, *769*, 289-304.

Estes, W. K. (1961). A descriptive approach to the dynamics of choice behavior. *Behavioral Science*, *6*, 177-184.

Falk, R., & Konold, C. (1997). Making sense of randomness: Implicit encoding as a basis for judgment. *Psychological Review*, *104*, 301-318.

Gazzaniga, M. S. (1989). Organization of the human brain. *Science*, *245*, 947-952.

Gazzaniga, M. S. (1995). Principles of human brain organization derived from split-brain studies. *Neuron*, *14*, 217-228.

Gopnik, A. (1998). Explanation as orgasm. *Minds and Machines*, *8*, 101-118.

Hinson, J. M., & Staddon, J. E. R. (1983). Matching, maximizing and hill-climbing. *Journal of the Experimental Analysis of Behavior*, *40*, 321-331.

Humphreys, L. G. (1939). Acquisition and extinction of verbal expectations in a situation analogous to conditioning. *Journal of Experimental Psychology, 25*, 294-301.

Kahneman, D., Slovic, P., & Tversky, A. (1982). Judgment under uncertainty: Heuristics and biases. New York: Cambridge University Press.

Lopes, L. L. (1982). Doing the impossible: A note on induction and the experience of randomness. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *8*, 626-636.

Lopes, L. L., & Oden, G. C. (1987). Distinguishing between random and nonrandom events. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *13*, 392-400.

Metcalfe, J., Funnell, M., & Gazzaniga, M. S. (1995). Right-hemisphere superiority: Studies of a split-brain patient. *Psychological Science*, *6*, 157-163.

Smith, E. E., Jonides, J, Koeppe, R. A., Awh, E., Schumacher, E. H., & Minoshima, S. (1995). Spatial versus object working memory: PET investigations. *Journal of Cognitive Neuroscience*, *7*, 337-356.

Wolford, G., Miller, M. B., & Gazzaniga, M. (2000). The left hemisphere's role in hypothesis formation. *Journal of Neuroscience*, *20*, RC64.

Yellott, J. I. Jr. (1969). Probability learning with noncontingent success. *Journal of Mathematical Psychology*, *6*, 541-575.

Sommaire

Les personnes qui observent une série d'événements semblent vouloir y déceler une tendance explicative qui les aiderait à prédire des événements ultérieurs. Elles consacreraient beaucoup de temps à la recherche de tendances même en présence d'une série aléatoire dont la nature leur a été signalée. Dans le présent article, nous offrons la preuve que des structures neurales de l'hémisphère gauche président à la recherche de tendances et que l'incompréhension du caractère aléatoire influence la propension à vouloir déceler des tendances. Nous faisons appel à un paradigme de probabilités arbitraires pour illustrer nos affirmations.

Les participants qui se fondent sur un paradigme de probabilités arbitraires prédisent lequel de deux événements se produira à chaque essai. Règle générale, ils se reportent à la fréquence (prédisent un événement donné selon la fréquence à laquelle il s'est produit lors d'essais antérieurs), bien que l'appariement par fréquence soit non optimal face à des séquences aléatoires. La stratégie optimale consiste à deviner sans exception l'événement le plus fréquent, en maximisant les résultats. Nous posons que l'appariement par fréquence découle de la recherche de tendances, même dans des séquences aléatoires. Wolford, Miller et Gazzaniga (2000) ont étudié à la fois des personnes qui avaient subi une calosotomie et d'autres dont le lobe frontal avait été atteint et ont constaté que l'ap-

pariement par fréquence se produisait dans l'hémisphère gauche, tandis que la maximisation se situait dans l'hémisphère droit. Nous démontrons, dans le présent article, qu'une tâche secondaire qui vise à mobiliser les ressources de l'hémisphère gauche (mémoire de travail verbale) pousse les participants à maximiser, tandis qu'une tâche qui fait appel à l'hémisphère droit (mémoire de travail spatiale) n'occasionne aucun changement à l'appariement par fréquence. Nos résultats correspondent à nos constatations antérieures, selon lesquelles l'appariement par fréquence dépend de structures de l'hémisphère gauche.

Les gens semblent interpréter erronément le carac-tère aléatoire. Invités à produire une séquence aléatoire de piles et de faces, ils retiennent trop d'alternances et trop peu de longues suites (Lopes et Oden, 1987). La seconde expérience a consisté à composer des séquences conformes aux attentes des gens quant à leur caractère aléatoire. En modifiant les probabilités conditionnelles, nous avons conçu des séquences dont les probabilités marginales étaient identiques à celles de séquences véritablement aléatoires, à ceci près qu'elles comportaient plus d'alternances et moins de longues suites. Les participants se sont rapprochés de la maximisation face aux séquences aux probabilités conditionnelles modifiées.